

Assignment of Chapter 2

1. A military commander is trapped in a fortress. His only available communication method to allied forces is a flock of carrier pigeons. Each pigeon can carry one word (8 bits). Pigeons are dispatched every 5 minutes, and each pigeon takes exactly 3 minutes to arrive at the destination.
 - (a) Assume all pigeons safely arrive at the destination. What is the capacity of this communication link, in bits per hour?
 - (b) Now assume enemies attempt to shoot down the pigeons, and a fraction α of dispatched pigeons are hit. Pigeons are released at a constant rate. The receiver knows when a pigeon fails to arrive. What is the capacity of this link?
 - (c) Now suppose the enemies become more sophisticated. Whenever they shoot down a pigeon, they release a fake pigeon carrying a random word, uniformly selected over all 8-bit words. What is the capacity of this communication link?

2. Consider a discrete memoryless channel as shown in the figure below.
 - (a) Please write down the transition probability matrix of this channel.
 - (b) Please determine the channel capacity.
 - (c) Please determine the input and output probability distributions when the channel operates at capacity.

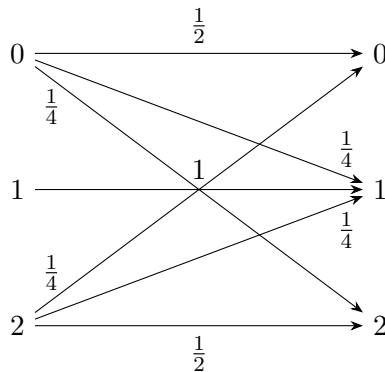


Figure 1: The trellis of the proposed channel.

3. Consider a Gaussian channel $Y_i = X_i + Z_i$ with a time-varying mean.

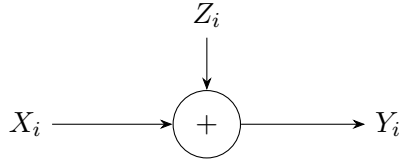


Figure 2: The additive Gaussian noise channel.

Let $Z_1, Z_2, \dots, Z_i$ be mutually independent Gaussian random variables, where $Z_i \sim \mathcal{N}(\mu_i, \sigma_0^2)$. The input X_i satisfies the average power constraint P . Please determine the channel capacity under each of the following conditions:

- (a) For all i , $\mu_i = 0$.
- (b) $\mu_i = e^i$, $i = 1, 2, \dots$, assume both the transmitter and receiver know μ_i .
- (c) Now consider a binary-input discrete memoryless channel (B-DMC) with additive noise. Let $X \in \{0, 1\}$, and the additive noise Z is independent of X , with $\Pr\{Z = 0\} = \Pr\{Z = a\} = 1/2$.